Correlation of Mass Transfer Data: Comments on an Article by Son and Hanratty

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Son and Hanratty (*9*) point out a disagreement about the proper functional form for empirical equations used to correlate heat and mass transfer data. They examined some experimental data available and came to the conclusion that the proper form is

$$<K_\infty^+> = 0.121\ N_{Sc}^{-3/4} \tag{1}$$

This result was checked by using both entrance region mass transfer data and mass transfer data from the fully developed transfer region. Equation (1) is equivalent to

$$N_{Sh} = 0.121\ N_{Re}\ f^{1/2}\ N_{Sc}^{1/4} \tag{2}$$

Others (*4, 6, 8, 10*) have recently come to the conclusion that the proper functional form is

$$N_{Sh} = a\ N_{Re}\ f^b\ N_{Sc}^{1/3} \tag{3}$$

The exponent for f is not definitely fixed.

Son and Hanratty did not quote the most recent data dealing with mass transfer in liquids, which seem applicable to their study. In Figure 1, data used by Son and Hanratty is plotted along with that of Hamilton (*3*) and Hubbard (*5*). These additional data show that Son and Hanratty probably do not use the best expression for the empirical correlation. Lines representing Equations (1) and (3) are also shown in Figure 1. The results are not really conclusive as to the proper Schmidt number dependence for $<K_\infty^+>$, except that in the range $10^4 < N_{Sc} < 10^5$, the data diverge noticeably from the correlating equation by Son and Hanratty. An equation containing $N_{Sc}^{-2/3}$ rather than $N_{Sc}^{-3/4}$ seems to represent the data better. An alternative interpretation would be that the exponent of N_{Sc}, Equations (2) and (3), is a function of Schmidt number taking the value 1/3 for $N_{Sc} > 10^4$ and the value ¼ for smaller values of N_{Sc}. This would mean that the limiting relation for eddy diffusivity described by Son and Hanratty (*9*) also depends on the Schmidt number. To make any conclusive statement about this will require more accurate and extensive data.

The friction velocity, u_*, has always been used as the reference velocity in discussing turbulent flow problems. Son and Hanratty use it to form the dimensionless mass transfer coefficient, $<K_\infty^+>$. There is no real experimental evidence that this is a good method for describing velocity profiles or mass transfer data. Rothfus and Monrad (*7*) show that v^+ is not a unique function of y^+ except at high Reynolds numbers. Below $N_{Re} = 20{,}000$, a Reynolds number dependence appears. The dimensionless mass transfer coefficient

$$<K_\infty^+> = \frac{k}{u_*} = \frac{k}{<v>\sqrt{\frac{f}{2}}} \tag{4}$$

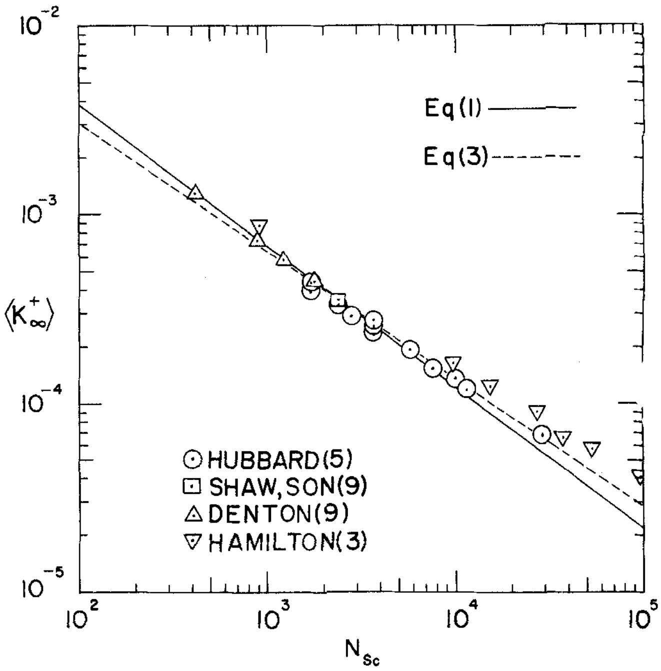

Fig. 1. Variation of the dimensionless mass transfer coefficient with Schmidt number.

is not a function of the Reynolds number and Son and Hanratty cite evidence to show this. The data of Hubbard (*5*) shows that within experimental accuracy, an alternative dimensionless mass transfer coefficient

$$<K_\infty^{++}> = \frac{k}{<v>\frac{f}{2}} \tag{5}$$

also does not depend on the Reynolds number. By choosing a dimensionless mass transfer coefficient of the form given in Equations (4) or (5), one arbitrarily selects the exponent of the friction factor in Equation (3). Most authors give this exponent a constant value of either 1 or ½. Hubbard and Lightfoot (*6*) point out that the exponent, b in Equation (3), may not be constant at all. No conclusive results about this have been obtained, though Harriot and Hamilton (*4*) come to the same conclusion. When more data is available, b can be determined as a function of Schmidt number, and it need not be assumed constant.

Hubbard's (5) mass transfer data, obtained from measurements made in a rectangular channel, brings up another point mentioned by Son and Hanratty. They felt that secondary flows in such channels would make the data inapplicable to their work. Results presented by Gessner (1) show that these secondary flow effects are confined largely to the corners and are probably negligible near the center of the channel. Gessner measured secondary flow velocities in air in channels having aspect ratios of 1 and 2. Hubbard measured mass transfer coefficients in an aqueous electrolyte in a channel having an aspect ratio of 6.5. The latter measurements were confined to the central 42.3% of the channel.

Conclusive statements cannot be made about the effect of secondary flows in this case, since secondary flow data for channels of aspect ratio larger than 2 are not available, but some general idea of the effect may be deduced by extrapolating Gessner's data. For a channel running full, the dimensionless flow patterns depend only on the Reynolds number, as fluid properties do not explicitly appear. Data given by Gessner is plotted in Figure 2 where dimensionless transverse velocity (secondary flow velocity) is shown as a function of Reynolds number. In Figure 3, these data are replotted to show the transverse velocity as a function of aspect ratio. The complete data are not plotted, since there is some ambiguity in the presentation of the data given in the thesis. There is also a disagreement between the data presented by Gessner (1) and the same data quoted by Gessner and Jones (2). In spite of this ambiguity, these data indicate that the secondary flow velocity is unlikely to be more than 1% of the axial center-line velocity, and that the secondary flow decreases toward the center of the channel. Thus, no effect greater than the experimental error is expected because of the presence of secondary flow in the channel. Mass transfer data obtained in rectangular channels should be applicable for testing the empirical correlations for dimensionless mass transfer coefficients.

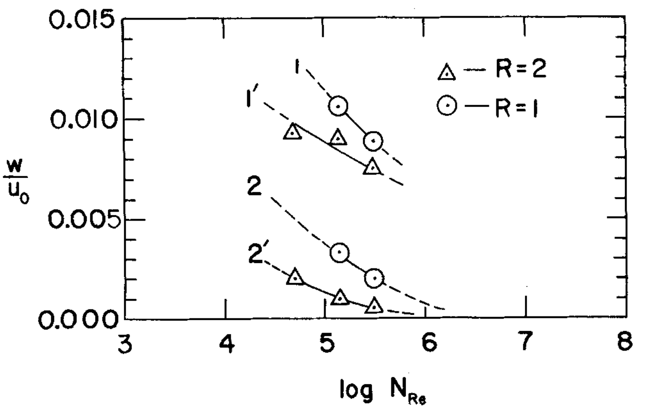

Fig. 2. Variation of secondary flow velocity with Reynolds number at $y = 0.022$ in. from the longer channel wall at different distances from the channel center line. Curves 1 and 1' are for $z = 0.75B$; curves 2 and 2' are for $z = 0.25B$. $z = 0$ is at the center line. Data reported by Gessner (1).

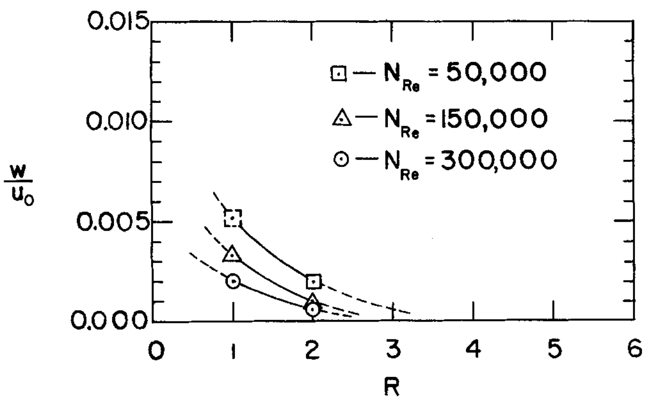

Fig. 3. Variation of secondary flow velocity with aspect ratio at $y = 0.022$ in. from the longer channel wall and $z = 0.25B$. Data reported by Gessner (1).

ACKNOWLEDGMENT

The correspondent benefited greatly from a discussion of this problem with Professor T. J. Hanratty.

NOTATION

a = constant, Equation (3)
b = parameter indicating friction factor dependence, Equation (3)
B = half width of the rectangular channel
D_e = $4\ R_h$
$\mathcal{D}_{im}$ = effective binary diffusion coefficient for the species participating in the electrode reaction
f = fanning friction factor
k = asymptotic mass transfer coefficient for the region where fully developed transfer occurs
$<K_x{}^+>$ = dimensionless mass transfer coefficient defined by Equation (4)
$<K_\infty{}^{++}>$ = dimensionless mass transfer coefficient defined by Equation (5)
N_{Re} = $D_e\rho<v>/\mu$ = Reynolds number
N_{Sc} = $\mu/\rho\mathcal{D}_{im}$ = Schmidt number
N_{Sh} = $k\ D_e/\mathcal{D}_{im}$ = Sherwood number
R = aspect ratio
R_h = hydraulic radius
u_0 = center-line velocity
u_* = $\sqrt{\tau_w/\rho}$ = friction velocity
$<v>$ = average velocity
v = local fluid velocity
v^+ = v/u_* = dimensionless velocity
y = distance from the solid boundary
y^+ = $y\ \rho u_*/\mu$ = dimensionless distance
w = transverse velocity in the conduit
z = transverse coordinate direction measured from center line
μ = fluid viscosity
ρ = fluid density
τ_w = wall shear stress

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